DOI 10.15507/2079-6900.27.202502.229-242 ISSN 2079-6900 (Print)
Original article ISSN 2587-7496 (Online)

MSC2020 76B15, 76B25, 35Q53

Asymptotic and numerical study to the damped Schamel equation

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Abstract. Analytical and numerical solutions of the damped Schamel equation, describing the dynamics of ion-acoustic waves in magnetized plasma, are presented. A small parameter is introduced in the equation before the dissipative term, ensuring that in its absence the solution reduces to a solitary wave (soliton). The asymptotic method employed for solving the equation is a variant of the Krylov-Bogolyubov-Mitropolsky multiple-scale technique. In the first-order approximation, the solution is described by a traveling solitary wave with slowly varying parameters. The second-order approximation yields the evolution laws for the soliton's amplitude and phase as functions of «slow» time. Additionally, exact integral conservation laws (mass and energy of the wave field), derived directly from the original damped Schamel equation, are utilized. These integrals allow estimating the soliton's radiative losses, particularly the mass of the so-called tail formed behind the soliton due to dissipation. Direct numerical solutions of the original equation, obtained via a pseudospectral method, confirm the asymptotic laws governing the soliton's amplitude decay caused by dissipation. Another limiting case – strong dissipation (dominant over nonlinearity and dispersion), is also investigated, demonstrating that the soliton decays as a linear impulse, which is validated numerically.

Keywords: ion-acoustic waves, Shamel equation, solitary wave, method of multiple scales, pseudospectral method

For citation: *M. V. Flamarion, E. N. Pelinovsky, T. G. Talipova.* Asymptotic and numerical study to the damped Schamel equation. *Zhurnal Srednevolzhskogo matematicheskogo obshchestva.* 27:2(2025), 229–242. DOI: 10.15507/2079-6900.27.202502.229-242

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Оригинальная статья

УДК 517.958

Асимптотическое и численное исследование уравнения Шамеля с затуханием

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Аннотация. Приведено аналитическое и численное решение модельного уравнения Шамеля с затуханием, описывающим динамику ионно-звуковых волн в замагниченной плазме. Малый параметр в уравнении введен перед диссипативным слагаемым, так что в его отсутствие решением уравнения Шамеля является уединенная волна (солитон). Для его решения применен асимптотический метод, являющийся разновидностью метода многих масштабов Крылова-Боголюбова-Митропольского. В первом приближении по малому параметру решение описывается уединенной бегущей волной с параметрами, медленно изменяющимися со временем. Во втором приближении находятся законы изменения амплитуды и фазы солитона, как функции «медленного» времени. Наряду с этим используются интегральные законы массы и энергии волнового поля, вытекающие точно из исходного модульного уравнения Шамеля с диссипацией. Показывается, что эти интегралы позволяют оценить величину излучения солитона, в частности, массу так называемого хвоста, возникающего за солитоном в процессе его диссипации. Прямое численное решение исходного уравнения псевдоспектральным методом подтвердило асимптотические законы изменения амплитуды солитона из-за его диссипации. Исследован также другой предельный случай сильной диссипации (по сравнению с нелинейностью и диссипацией), когда солитон затухает как линейный импульс, этот процесс подтвержден численно.

Ключевые слова: ионно-звуковые волны, уравнение Шамеля, уединенная волна, метод многих масштабов, псевдоспектральный метод

Для цитирования: Фламарион М. В., Пелиновский Е. Н., Талипова Т. Г. Асимптотическое и численное исследование уравнения Шамеля с затуханием // *Журнал Средневолжского математического общества.* 2025. Т. 27, № 2. С. 229-242. DOI: 10.15507/2079-6900.27.202502.229-242

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1. Introduction

The Schamel equation was first introduced in the study of undamped electrostatic waves in a Maxwellian plasma [1], [2]. Its analytical solutions in the form of solitary waves taking into account the forcing are obtained in [3], [4]. The interactions between ions and electrons, considering various factors like distribution functions, plasma density, temperature gradients, and collisions are analyzed in [5]-[8]. The same equation describe the nonlinear wave dynamics of cylindrical shells [9]. Unlike the well-known Korteweg–de Vries (KdV) equation, the Schamel equation incorporates a modular term that modifies the representation of nonlinearity. This crucial difference renders the Schamel equation nonintegrable, introducing mathematical challenges due to the non-analytic nature of the function–challenges that do not arise in integrable equations commonly encountered in plasma physics, such as the modified KdV (mKdV) equation and the Gardner equation [1], [2], [10], [11]. The Schamel equation allows for solitary wave solutions of both polarities, but because of its nonintegrability, their interactions are inelastic, resulting in a small-amplitude dispersive tail that forms immediately after two solitary waves collide [12], [13]. Flamarion et al. [14] extended this analysis to an ensemble, examining the interactions of multiple solitary waves with random phases. They demonstrated that the dispersive tails generated during each collision can act as a mechanism for the formation of freak waves.

The interaction between solitary waves and an external force has also been explored within the framework of the Schamel equation. Chowdhury et al. [4] derived a Schamel-type equation that accounts for the presence of an external force and investigated the interaction between a solitary wave and a time-dependent external periodic force, using both asymptotic analysis and numerical simulations. Later, Flamarion and Pelinovsky [15] studied the effects of a spatially dependent force on solitary waves, identifying conditions for resonance between the external force and the solitary waves through asymptotic and numerical methods. Since its derivation in 1972, the Schamel equation continues to present intriguing challenges and remains a vibrant area of research.

Shan [16] explored the nonlinear characteristics of high-frequency electron-acoustic (EA) in a dissipative plasma, consisting of a cold beam electron fluid, Schamel-kappa distributed hot trapped electrons, and stationary ions. Using the multiple scale expansion method, Shan derived a damped Schamel equation to describe small-amplitude electrostatic potential disturbances, incorporating dissipative effects. This damped equation was later employed by Sultana and Kourakis [17] in their study of the electrostatic potential, where they analyzed the nonlinear properties of dissipative ion-acoustic solitary waves in the presence of trapped electrons. Although numerical solutions were obtained in both studies, their mathematical properties were not thoroughly investigated.

The goal of this work is to examine the damped Schamel equation derived in [16], [17] and obtain an asymptotic solution by assuming that a solitary wave undergoes adiabatic evolution. We demonstrate that a tail forms behind the solitary wave with negative mass. The asymptotic results are then compared with direct numerical simulations, revealing good qualitative agreement.

The article is structured as follows: Section 2 presents the canonical damped Schamel equation, while Section 3 covers the asymptotic results. Numerical results are discussed in Section 4, and final conclusions are provided in Section 5.

2. The damped Schamel equation

As mentioned in the introduction, the damped Schamel equation has appeared in the literature in various contexts. In this work, we focus on investigating the evolution of solitary wave solutions in the presence of damping. To achieve this, we consider the Schamel equation in its canonical form

$$u_t + \sqrt{|u|}u_x + u_{xxx} + \epsilon\nu u = 0. \tag{2.1}$$

In this equation, $u(x,t)$ represents the wave field at position x and time t, ν denotes the damping coefficient, and ϵ is a small positive parameter that characterizes the strength of the viscosity. In the absence of damping, the Schamel equation(2.1) admits solitary wave solutions, which can be expressed as follows

$$u(x,t) = a\,\mathrm{sech}^4\Big(k(x-ct)\Big), \text{ where } c = \frac{8\sqrt{|a|}}{15}, \quad k = \sqrt{\frac{c}{16}}. \tag{2.2}$$

Here, a denotes the solitary wave amplitude, which can also take on negative values, c represents the solitary wave speed, and k characterizes the solitary wave wavenumber.

The damped Schamel equation has two important quantities: the mass and momentum associated with equation (2.1) are, respectively,

$$M(t) = \int_{-\infty}^{+\infty} u(x,t)dx \quad \text{and} \quad E(t) = \int_{-\infty}^{+\infty} u^2(x,t)dx.$$

The momentum equation is

$$\frac{dE}{dt} = -2\epsilon\nu E(t), \tag{2.3}$$

which has solution $E(t) = E_0 e^{-2\epsilon\nu t}$, where E_0 stands for the initial momentum. Meanwhile the mass balance equation

$$\frac{dM}{dt} = -\epsilon\nu M(t), \tag{2.4}$$

which has solution

$$M(t) = M_0 e^{-\epsilon\nu t}. \tag{2.5}$$

Here, M_0 represents the initial mass.

3. Asymptotical solitary wave solution of the damped Schamel equation

When the damping term is introduced into the Schamel equation, solitary wave solutions of the form (2.2) cease to exist. However, if the damping is weak, it is reasonable to expect that the solitary wave might nearly preserve its main characteristics, such as amplitude, speed, and width, over short periods. In other words, it is natural to assume that the solitary wave undergoes an adiabatic transformation [18]-[20].

We assume that the solitary wave is expressed asymptotically in the slowly varying time scale $T = \epsilon t$

$$u(\mathbf{\Phi}, T) = a(T)\mathrm{sech}^4\Big(k(T)\mathbf{\Phi}\Big), \quad \mathbf{\Phi} = x - X(T), \quad X(T) = x_0 + \frac{1}{\epsilon}\int_0^T c(T)dT,$$

where $a(T)$ is the modulated amplitude, $X(T)$ is the solitary wave crest position and x_0 its phase, $c(T)$ is the variable speed and $k(T)$ the typical variable solitary wave width. Here, x_0 is the phase, a is the modulated amplitude and c is the speed. The wave field is given by the asymptotic expansion [21]-[23]

$$u(\mathbf{\Phi}, T) = u_0 + \epsilon u_1 + \epsilon^2 u_2 + \cdots,$$
$$c(T) = c_0 + \epsilon c_1 + \epsilon^2 c_2 + \cdots.$$

The first order solutions, u_0 and c_0 are directly determined through formula (2.2). We consider a solitary wave with positive polarity ($a > 0$). In the first order ϵ, we have

$$Lu_1 \equiv -c_0\frac{\partial u_1}{\partial \mathbf{\Phi}} + \frac{\partial}{\partial \mathbf{\Phi}}\Big[f'(u_0)u_1\Big] + \frac{\partial^3 u_1}{\partial \mathbf{\Phi}^3} = F_1 \equiv -\nu u_0 - \frac{\partial u_0}{\partial T} + c_1\frac{\partial u_0}{\partial \mathbf{\Phi}},$$

where $f(u) = u|u|^{1/2}$. The operator L is a self-adjoint operator and

$$L\xi = 0 \Longrightarrow \xi = u_0,$$

is the only bounded solution. The compatibility condition is

$$\int_{-\infty}^{+\infty} F_1 \xi d\mathbf{\Phi} = 0 \Longrightarrow \int_{-\infty}^{+\infty} F_1 u_0 d\mathbf{\Phi} = 0$$

Consequently we have that

$$\int_{-\infty}^{+\infty} u_0\frac{\partial u_0}{\partial T}d\mathbf{\Phi} = -\nu\int_{-\infty}^{+\infty} u_0^2 d\mathbf{\Phi}.$$

In other words,

$$\frac{1}{2}\frac{d}{dT}\int_{-\infty}^{+\infty} u_0^2 d\mathbf{\Phi} = -2\nu\int_{-\infty}^{+\infty} u_0^2 d\mathbf{\Phi}.$$

Using the fact that $T = \epsilon t$ we obtain

$$\frac{1}{2}\frac{d}{dt}\int_{-\infty}^{+\infty} u_0^2 d\mathbf{\Phi} = -2\epsilon\nu\int_{-\infty}^{+\infty} u_0^2 d\mathbf{\Phi}. \tag{3.1}$$

Notice that equations and (2.3) and (3.1) are the same, it means that tails does not contribute in the momentum in the first-order approximation. As a result, the momentum can be computed analytically as follows

$$E(T) = \frac{a^2(T)}{2}\int_{-\infty}^{+\infty}\mathrm{sech}^8(k\mathbf{\Phi})d\mathbf{\Phi} = \frac{a^2(T)}{2k}\int_{-\infty}^{+\infty}\mathrm{sech}^8(\mathbf{\Phi})d\mathbf{\Phi} = \frac{16}{35}\sqrt{30}\,a^{7/4}(T). \tag{3.2}$$

Substituting equation (3.2) in 2.3 leads to the following ordinary differential equation for the solitary wave amplitude

$$\frac{da}{dt} = -\frac{8}{7}\epsilon\nu a(t). \tag{3.3}$$

Consequently,

$$a(t) = a_0 e^{-\frac{8}{7}\epsilon\nu t}. \tag{3.4}$$

It is important to note that the asymptotic procedure does not account for the mass balance. This occurs because the contribution of the tail is neglected in the asymptotic expansion. Consequently, the total mass of the wave field should be considered as comprising two components: the mass contribution to the solitary wave and the mass contribution to its tail, which are separated as follows

$$M(t) = M_s(t) + M_t(t),$$

where M_s represents the mass contribution of the solitary wave and M_t accounts for the tail mass. The tail mass can be computed in terms of total mass and solitary wave mass as

$$M_t(t) = M(t) - M_s(t),$$

where $M(t)$ is give in equation (2.5). On the other hand, the solitary wave mass is

$$M_s(T) = \frac{a(T)}{2}\int_{-\infty}^{+\infty} \operatorname{sech}^4(k\mathbf{\Phi})d\mathbf{\Phi} = \frac{a(T)}{k}\int_{-\infty}^{+\infty} \operatorname{sech}^4(\mathbf{\Phi})d\mathbf{\Phi} = \frac{4}{3}\sqrt{30}a^{3/4}(T).$$

Substituting (3.4) into (2.4) that the solitary wave mass is

$$M_s(t) = \frac{4}{3}\sqrt{30}\, a_0^{3/4} e^{-\frac{6}{7}\epsilon\nu t}$$

Now, notice that our initial data is a solitary wave, thus the initial mass

$$M_0 = M_s(0) = \frac{4}{3}\sqrt{30}\, a_0^{3/4}.$$

Thus the tail mass is given by

$$M_t(t) = M(t) - M_s(t) = \frac{4}{3}\sqrt{30}\, a_0^{3/4} e^{-\epsilon\nu t} - \frac{4}{3}\sqrt{30}\, a_0^{3/4} e^{-\frac{6}{7}\epsilon\nu t} =$$
$$= \frac{4}{3}\sqrt{30}\, a_0^{3/4} e^{-\epsilon\nu t}(1 - e^{\frac{1}{7}\epsilon\nu t}). \tag{3.5}$$

This expression shows that for the total mass can be negative. Physically, this means that a tail forms just behind the solitary waves. Figure 3.1 displays the evolution of the ratio of the tails mass and the initial solitary wave mass.

4. Numerical simulation of the damped Schamel equation

We solve equation (2.1) using the standard pseudospectral method detailed in [24]. Spatial derivatives are computed spectrally and then equation is integrated over time using the classical explicit forth-order Runge-Kutta method. The same method has been employed to solve equation (2.1) in [14], [25], [26].

Figure 4.1 illustrates the evolution of a solitary wave in a weakly damped wave field. As the solitary wave propagates to the right, a tail forms behind it. The effect of damping is evident in the solitary wave amplitude, which decreases over time. The tail exhibits oscillatory behavior with negative mass. A series of snapshots capturing this evolution is

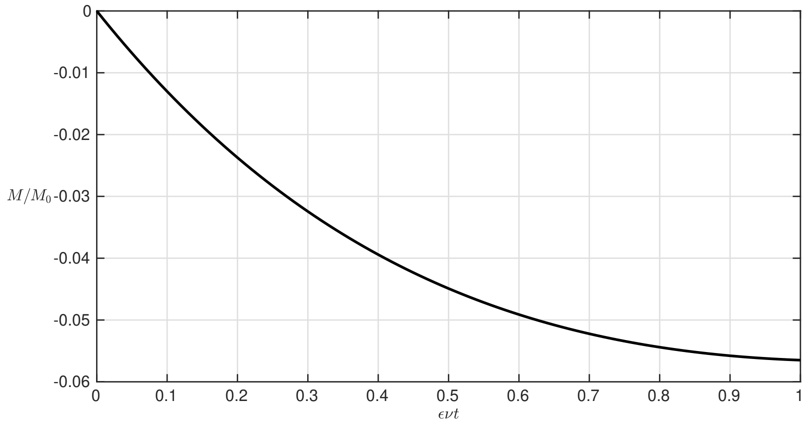


Рис. 3.1. Временная эволюция массы хвостов солитонного решения

Fig. 3.1. The evolution of the tail mass quantity

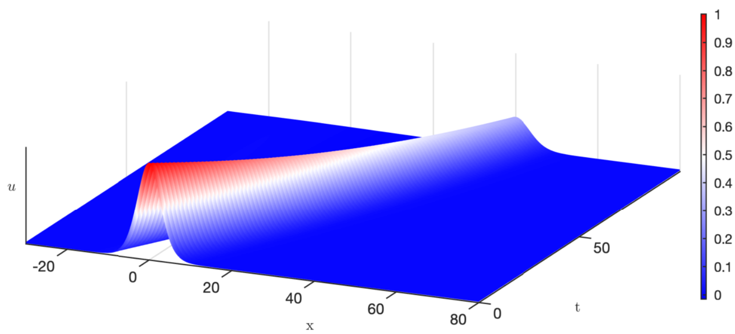


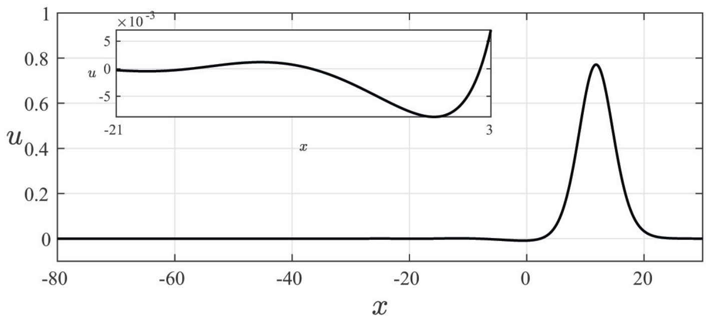
Рис. 4.1. Эволюция солитонного решения в среде со слабым затуханием ($\epsilon\nu = 0.01$)

Fig. 4.1. The evolution of a solitary wave in the weak damping field. Here, $\epsilon\nu = 0.01$

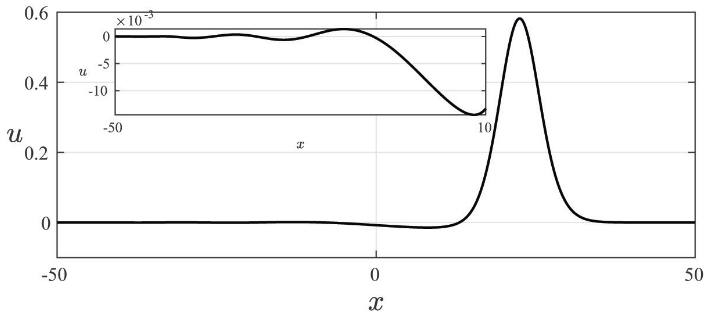
provided in Figure 4.2. The tail, which is of order $\mathcal{O}(10^{-4})$, is small compared to the solitary wave amplitude at early times. The wave field in the vicinity of $u = 0$ is smooth, but the nonlinearity here is not analytic. This is because, in the opposite case, the third derivative would have a jump.

In the case of strong damping, the solitary wave amplitude decays more rapidly, and the tail forms more quickly than in the previous scenario. Notably, the amplitude of the dispersive tail is larger under strong damping. Our simulations indicate that the tail amplitude scales with $\mathcal{O}(\nu^2)$. Figure 4.3 shows the evolution of a solitary wave in a stronger damping field, where a tail forms behind the solitary wave. For further details, see Figure 4.4. Despite the quantitative differences with higher damping values, the qualitative features remain consistent.

As we increase further the damping coefficient to $\nu = 1$. In this case, the damping is so strong the the solitary wave is destroyed in a short period of time. An example of such case



a) $t = 25$



b) $t = 50$

Рис. 4.2. Моментальные снимки эволюции солитонного решения. Начальная амплитуда уединённой волны $a_0 = 1$, коэффициент затухания $\epsilon\nu = 0.01$.

Fig 4.2. Snapshots of the solitary wave evolution. The initial solitary wave amplitude is $a_0 = 1$ and the damping coefficient $\epsilon\nu = 0.01$.

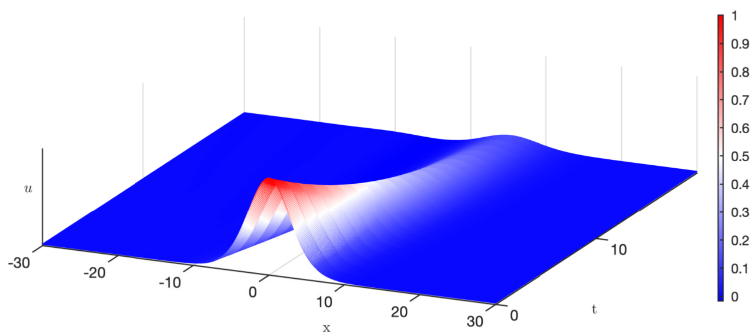
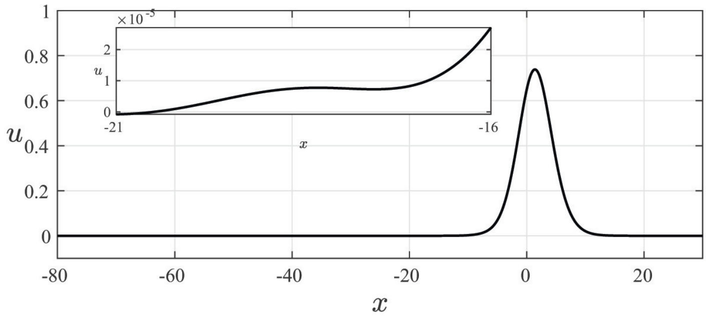


Рис. 4.3. Эволюция солитонного решения в среде с затуханием ($\epsilon\nu = 0.1$)

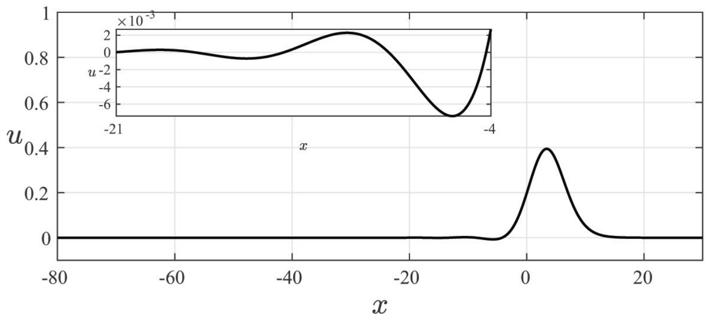
Fig. 4.3. The evolution of a solitary wave in the damping field. Here, $\epsilon\nu = 0.1$

is shown in Figure 4.5.

Finally, Figure 4.6 compares the amplitude of the solitary wave computed asymptotically



$a)\ t = 4$



$b)\ t = 10$

Рис. 4.4. Моментальные снимки эволюции солитонного решения. Начальная амплитуда уединённой волны $a_0 = 1$, коэффициент затухания $\epsilon\nu = 0.1$.

Fig 4.4. Snapshots of the solitary wave evolution. The initial solitary wave amplitude is $a_0 = 1$ and the damping coefficient $\epsilon\nu = 0.1$.

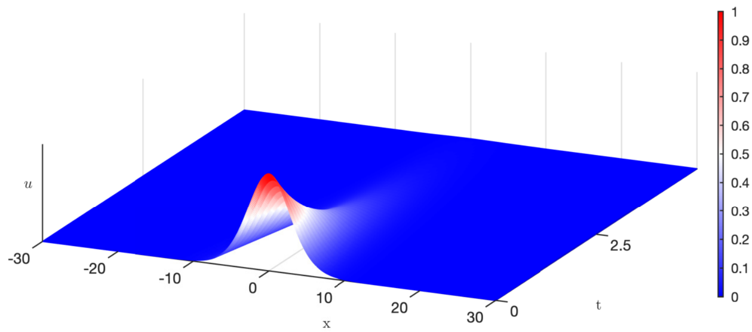
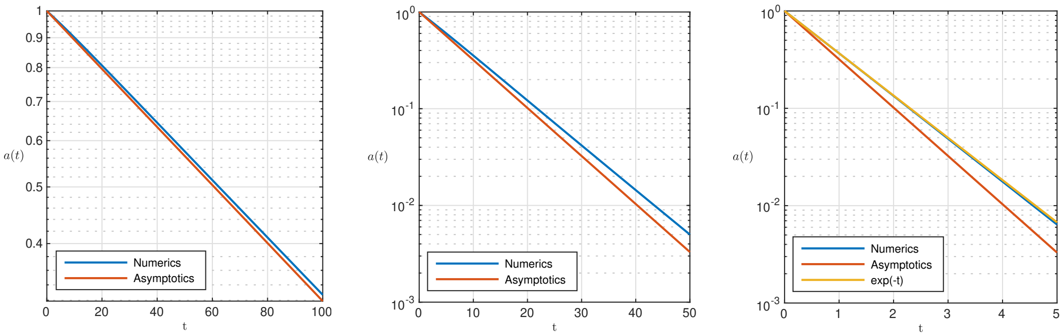


Рис. 4.5. Эволюция солитонного решения в среде с сильным затуханием ($\epsilon\nu = 1$)

Fig. 4.5. The evolution of a solitary wave in the strong damping field. Here, $\epsilon\nu = 1$

with that obtained from numerical simulations for different values of ν. Both approaches yield qualitatively similar results. However, the quantitative agreement is observed only at early times, with discrepancies emerging as time progresses. Notice yet that when damping is stronger, the solitary wave cannot maintain its shape or behave as a pulse of constant width, as the effects of nonlinearity and dispersion become smaller compared to damping. In this scenario, the energy balance (or the direct Schamel equation without nonlinearity and dispersion) results in $a(t) \sim \exp(-\epsilon\nu t)$. Consequently, the slope of the curves in Figure 4.6 (right) changes from 8/7 for weak damping to 1 for strong damping, indicating a slower rate of decay.



a) $\epsilon\nu = 0.01$ *b*) $\epsilon\nu = 0.1$ *c*) $\epsilon\nu = 1$

Рис. 4.6. Сравнение амплитуд солитонных решений, полученных с помощью асимптотической теории, с результатами численного моделирования в полулогарифмическом масштабе.

Fig 4.6. Comparison between the solitary wave amplitudes predicted by the asymptotic theory and the numerical simulations in the semi-log scale.

5. Conclusion

In the framework of the Schamel equation, we asymptotically determined the solitary wave amplitude and its decay rate, as well as the solitary wave's position at any given time. Additionally, we demonstrated that a tail forms behind the solitary wave and calculated its mass, showing that the tail mass is negative. To validate the asymptotic results, we performed numerical simulations. Overall, there is good qualitative agreement between the asymptotic predictions and the numerical results for weak and strong damping.

Funding. Research of EP and TT was supproted by the RSF grant 24-47-02007.

Acknowledgements. The authors are grateful to Ioann Melnikov for his assistance in preparing the LaTeX version of this paper.

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Submitted 27.11.2024; Revised 25.04.2025; Accepted 28.05.2025

The authors have read and approved the final manuscript.
Conflict of interest: The authors declare no conflict of interest.

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Поступила 27.11.2024; доработана после рецензирования 25.04.2025; принята к публикации 28.05.2025

Авторы прочитали и одобрили окончательный вариант рукописи.
Конфликт интересов: авторы заявляют об отсутствии конфликта интересов.