DOI 10.15507/2079-6900.25.202301.542-553
Original article

ISSN 2079-6900 (Print)
ISSN 2587-7496 (Online)

MSC2020 45K05

Exact Solutions of One Nonlinear Countable-Dimensional System of Integro-Differential Equations

A. E. Rassadin

Higher School of Economics (Nizhny Novgorod, Russian Federation)

Abstract. In the present paper, a nonlinear countable-dimensional system of integro-differential equations is investigated, whose vector of unknowns is a countable set of functions of two variables. These variables are interpreted as spatial coordinate and time. The nonlinearity of this system is constructed from two simultaneous convolutions: first convolution is in the sense of functional analysis and the second one is in the sense of linear space of double-sided sequences. The initial condition for this system is a double-sided sequence of functions of one variable defined on the entire real axis. The system itself can be written as a single abstract equation in the linear space of double-sided sequences. As the system may be resolved with respect to the time derivative, it may be presented as a dynamical system. The solution of this abstract equation can be interpreted as an approximation of the solution of a nonlinear integro-differential equation, whose unknown function depends not only on time, but also on two spatial variables. General representation for exact solution of system under study is obtained in the paper. Also two kinds of particular examples of exact solutions are presented. The first demonstrates oscillatory spatio-temporal behavior, and the second one shows monotone in time behavior. In the paper typical graphs of the first components of these solutions are plotted. Moreover, it is demonstrated that using some procedure one can generate countable set of new exact system's solutions from previously found solutions. From radio engineering point of view this procedure just coincides with procedure of upsampling in digital signal processing.

Keywords: Cauchy problem, generating function, Laurent expansion, Fourier transform, Bessel functions of the first kind, modified Bessel functions

For citation: *A. E. Rassadin.* Exact Solutions of One Nonlinear Countable-Dimensional System of Integro-Differential Equations. *Zhurnal Srednevolzhskogo matematicheskogo obshchestva.* 25:1(2023), 542–553. DOI: https://doi.org/10.15507/2079-6900.25.202301.542-553

About the author:
Alexander E. Rassadin, Post-graduate student, National Research University — Higher School of Economics in Nizhny Novgorod (25/12, Bolshaya Pecherskaya Str., Nizhny Novgorod, 603155, Russia), ORCID: https://orcid.org/0000-0001-5644-4012, brat_ras@list.ru

Оригинальная статья

УДК 517.968.74

Точные решения одной нелинейной счётномерной системы интегро-дифференциальных уравнений

А. Э. Рассадин

Национальный исследовательский университет — Высшая школа экономики в Нижнем Новгороде (г. Нижний Новгород, Российская Федерация)

Аннотация. В представленной статье исследуется нелинейная счётномерная система интегро-дифференциальных уравнений, вектором неизвестных у которой является счётное множество функций двух переменных. Эти переменные интерпретируются как пространственная координата и время. Нелинейность рассматриваемой системы сконструирована из двух одновременных свёрток, а именно, из свёртки в смысле функционального анализа и из свёртки в смысле линейного пространства двусторонних последовательностей. Начальное условие для этой системы является двусторонней последовательностью функций одного переменного, определённых на всей действительной оси. Сама система может быть записана в виде одного абстрактного уравнения в линейном пространстве двусторонних последовательностей, разрешённого относительно производной по времени, то есть как динамическая система. Решение этого абстрактного уравнения можно трактовать как аппроксимацию решения нелинейного интегро-дифференциального уравнения, неизвестная функция которого зависит не только от времени, но и от двух пространственных переменных. В работе найдено общее представление точного решения исследуемой системы. Также даны два типа конкретных примеров точных решений этой системы. Первый из них демонстрирует пространственно-временное поведение колебательного характера, а второй тип решений ведёт себя во времени монотонно. В статье приведены типичные графики первых компонент этих решений. Более того, показано, что из этих точных решений в рамках некоторой процедуры можно сгенерировать счётное множество новых точных решений рассматриваемой системы. С точки зрения радиотехники эта процедура совпадает с процедурой повышения частоты дискретизации в цифровой обработке сигналов.

Ключевые слова: задача Коши, производящая функция, ряд Лорана, преобразование Фурье, функции Бесселя первого рода, модифицированные функции Бесселя

Для цитирования: Рассадин А. Э. Точные решения одной нелинейной счётномерной системы интегро-дифференциальных уравнений // Журнал Средневолжского математического общества. 2023. Т. 25, № 1. С. 542-553. DOI: https://doi.org/10.15507/2079-6900.25.202301.542-553

Об авторе:

Рассадин Александр Эдуардович, аспирант, Национальный исследовательский университет — Высшая школа экономики в Нижнем Новгороде (603155, Россия, г. Нижний Новгород, ул. Большая Печерская, д. 25/12), ORCID: https://orcid.org/0000-0001-5644-4012, brat_ras@list.ru

1. Introduction

Integro-differential equations is known often to arise in physical kinetics [1], hydrodynamics [2], viscoelasticity [3], biology [4] etc. Currently, the theory of solving both nonlinear integro-differential equations and their systems is quite developed (see [5] and references therein). However, in all examples of such systems in this book, the number of unknown functions is a finite one, because of theory of systems of nonlinear integro-differential equations with denumerable set of unknown functions deals only with some quite abstract theorems [6], [7]. On the other hand it is obvious that presentation of a number of exact solutions of some nonlinear countable-dimensional systems of integro-differential equations stimulates the development of the theory of these systems.

In this article the following nonlinear countable-dimensional system of integro-differential equations is under consideration:

$$\frac{\partial u_n(x,t)}{\partial t} + \sum_{k=-\infty}^{+\infty} \int_{-\infty}^{+\infty} u_k(x-\xi,t)\, u_{n-k}(\xi,t)\, d\xi = 0\,, \quad n \in \mathbb{Z}\,, \tag{1.1}$$

where $\{u_n(x,t)\}_{n=-\infty}^{n=+\infty}$ is denumerable set of unknown functions.

System (1.1) is provided by the next denumerable set of initial conditions:

$$u_n(x,0) = u_n^0(x)\,, \quad x \in \mathbb{R}\,. \tag{1.2}$$

In this work countable set of exact solutions $\{u_n(x,t)\}_{n=-\infty}^{n=+\infty}$ of the Cauchy problem (1.1) – (1.2) has been constructed by means of technique of generating functions.

The rest of the article is organized as follows: in section 2 the procedure of construction of general representation of exact solution of the Cauchy problem (1.1) – (1.2) has been described. Section 3 deals with concrete examples of exact solutions of input nonlinear countable-dimensional system of integro-differential equations. Final section contains discussion of obtained results and of perspectives of further investigations.

2. General solution of the system

For the Cauchy problem (1.1) – (1.2) the following statement is valid.

Theorem 2.1. *General representation of exact solution of the Cauchy problem (1.1) – (1.2) is equal to:*

$$u_n(x,t) = \oint_{C_\rho} \int_{-\infty}^{+\infty} \frac{\tilde{U}^0(z;k)}{1 + t\,\tilde{U}^0(z;k)}\, \frac{\exp(i\,k\,x)}{z^{n+1}}\, \frac{dk}{2\,\pi}\, \frac{dz}{2\,\pi\,i} \tag{2.1}$$

where

$$\tilde{U}^0(z;k) = \int_{-\infty}^{+\infty} U^0(z;x)\, \exp(-i\,k\,x)\, dx \tag{2.2}$$

is the Fourier transform from the generating function of its initial condition:

$$U^0(z;x) = \sum_{n=-\infty}^{+\infty} u_n^0(x)\, z^n\,, \tag{2.3}$$

integration along the circle $C_\rho = \{z \in \mathbb{C} \mid \mid z \mid = \rho\}$ being counter clockwise.

First of all let us introduce for solution $\{u_n(x,t)\}_{n=-\infty}^{n=+\infty}$ of system (1.1) the generating function:

$$U(z;x,t) = \sum_{n=-\infty}^{+\infty} u_n(x,t)\, z^n\,. \tag{2.4}$$

Using definition (2.4) it is easy to find that this Cauchy problem can be rewritten as follows:

$$\frac{\partial U(z;x,t)}{\partial t}+\int_{-\infty}^{+\infty}U(z;x-\xi,t)\,U(z;\xi,t)d\xi=0\,,\quad U(z;x,0)=U^0(z;x)\,. \tag{2.5}$$

To solve the Cauchy problem (2.5) let one consider the Fourier transform of the generating function (2.4):

$$\tilde{U}(z;k,t)=\int_{-\infty}^{+\infty}U(z;x,t)\exp(-i\,k\,x)dx\,. \tag{2.6}$$

Further it is obvious that for the new unknown function (2.6) the Cauchy problem (2.5) is reduced to the next one:

$$\frac{\partial\tilde{U}(z;k,t)}{\partial t}+\tilde{U}^2(z;k,t)=0\,,\qquad \tilde{U}(z;k,0)=\tilde{U}^0(z;k)\,. \tag{2.7}$$

Exact solution of the Cauchy problem (2.7) is equal to:

$$\tilde{U}(z;k,t)=\frac{\tilde{U}^0(z;k)}{1+t\,\tilde{U}^0(z;k)}, \tag{2.8}$$

hence one can determine spatiotemporal evolution of the generating function (2.4) from formula (2.8) by means of the inverse Fourier transform:

$$U(z;x,t)=\int_{-\infty}^{+\infty}\frac{\tilde{U}^0(z;k)}{1+t\,\tilde{U}^0(z;k)}\,\exp(i\,k\,x)\,\frac{dk}{2\,\pi}\,. \tag{2.9}$$

At last calculating coefficients of the Laurent series from expression (2.9) one can establish the statement of the theorem.

3. Examples of exact solutions of the system

In practice instead of application of formula (2.1) it is more suitable to derive the Laurent expansion (2.4) and to extract exact solution $\{u_n(x,t)\}_{n=-\infty}^{n=+\infty}$ of the input Cauchy problem (1.1) – (1.2) from formula (2.9) straightforwardly. Two examples of such tricks are presented below.

3.1. Exact solution with oscillatory behavior

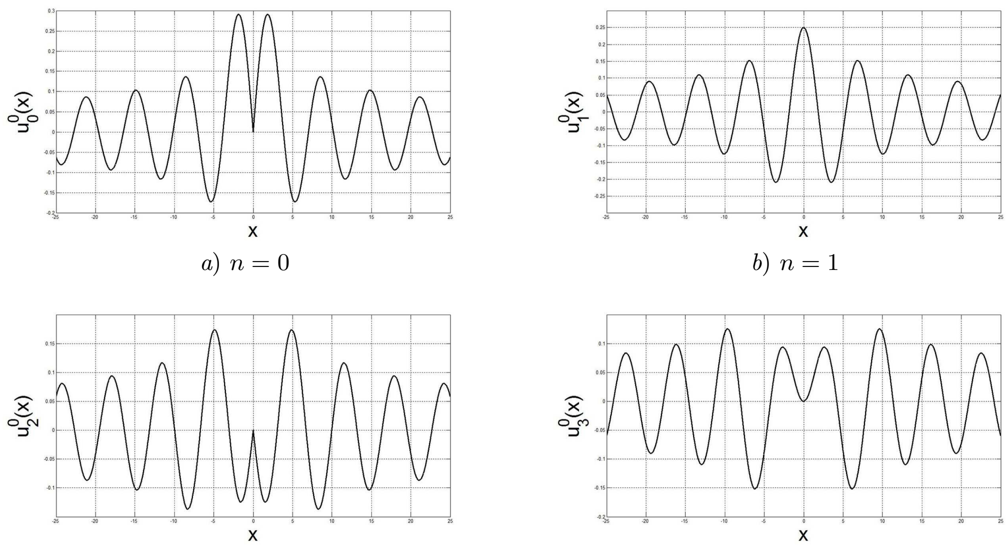
Let one consider the following vector $\{u_n^0(x)\}_{n=-\infty}^{n=+\infty}$ of initial conditions:

$$u_n^0(x)=(-1)^n\frac{A_0}{4\,a_0}\left[J_{n+1}\left(\frac{|x|}{a_0}\right)-J_{n-1}\left(\frac{|x|}{a_0}\right)\right],\quad n\in\mathbb{N}\,,$$

$$u_0^0(x)=\frac{A_0}{2\,a_0}\,J_1\left(\frac{|x|}{a_0}\right),\quad u_{-n}^0(x)=(-1)^n u_n^0(x), \tag{3.1}$$

where $J_n(\zeta)$ are Bessel functions of the first kind and A_0 , $a_0>0$.

Typical graphs of the first functions (3.1) are presented on Fig. 3.1.



a) $n = 0$ *b)* $n = 1$ *c)* $n = 2$ *d)* $n = 3$

Рис. 3.1. Графики первых функций $u_n^0(x)$ при $A_0 = 1$ и $a_0 = 1$ (колебательный режим): *a)* $n = 0$; *b)* $n = 1$; *c)* $n = 2$; *d)* $n = 3$

Fig. 3.1. Graphs of the first functions $u_n^0(x)$ under $A_0 = 1$ and $a_0 = 1$ (oscillatory behavior): *a)* $n = 0$; *b)* $n = 1$; *c)* $n = 2$; *d)* $n = 3$

The next theorem proves to be true.

Theorem 3.1. *Exact solution of the Cauchy problem* (1.1) – (1.2) *with initial conditions* (3.1) *is equal to ($n \in \mathbb{N}$):*

$$u_n(x,t) = \frac{(-1)^n A_0}{4\,a_0\sqrt{1+A_0 t}}\left[J_{n+1}\left(\frac{\sqrt{1+A_0 t}\,|x|}{a_0}\right) - J_{n-1}\left(\frac{\sqrt{1+A_0 t}\,|x|}{a_0}\right)\right],$$

$$u_0(x,t) = \frac{A_0}{2\,a_0\sqrt{1+A_0 t}}\,J_1\left(\frac{\sqrt{1+A_0 t}\,|x|}{a_0}\right), \qquad u_{-n}(x,t) = (-1)^n u_n(x,t)\,. \tag{3.2}$$

Substituting expressions (3.1) into the definition (2.3) of the generating function of initial condition for the Cauchy problem (1.1) – (1.2) and using the well-known Laurent series for a generating function of Bessel functions with integer order [8]:

$$\exp\left[\frac{\zeta}{2}\left(z - \frac{1}{z}\right)\right] = \sum_{n=-\infty}^{+\infty} J_n(\zeta)\,z^n\,, \tag{3.3}$$

one can find that:

$$U^0(z;x) = \frac{A_0}{4\,a_0}\left(z - \frac{1}{z}\right)\exp\left[-\frac{|x|}{2\,a_0}\left(z - \frac{1}{z}\right)\right]. \tag{3.4}$$

In accordance with formula (2.2) the Fourier transform of function (3.4) is equal to:

$$\tilde{U}^0(z;k) = A_0\left[1 + k^2\left(\frac{2\,a_0\,z}{z^2-1}\right)^2\right]^{-1}. \tag{3.5}$$

Inserting expression (3.5) into formula (2.8) it is easy to calculate that temporal evolution of the Fourier image of the generating function (2.4) is described as follows:

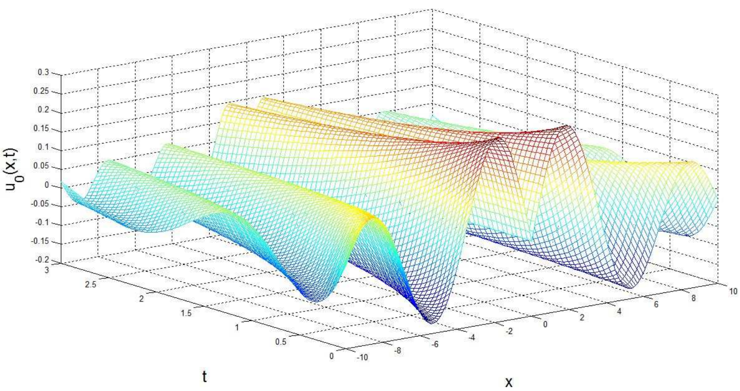
$$\tilde{U}(z;k,t) = A_0 \left[1 + A_0\,t + k^2 \left(\frac{2\,a_0\,z}{z^2-1}\right)^2\right]^{-1}. \qquad (3.6)$$

Further the inverse Fourier transform of function (3.6) represents temporal evolution of the generating function (2.4) itself:

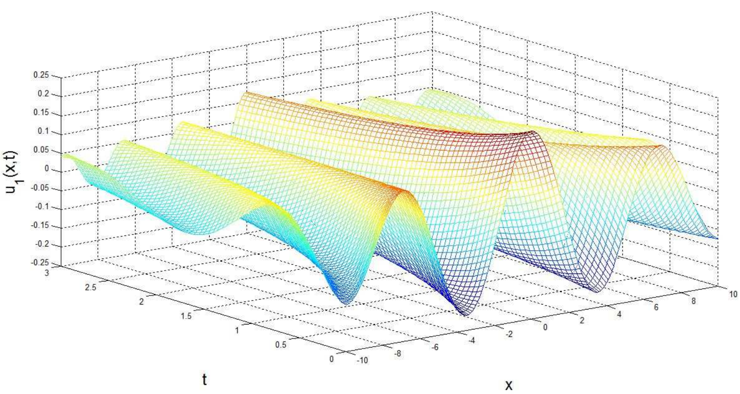
$$U(z;x,t) = \frac{A_0}{4\,a_0\,\sqrt{1+A_0 t}}\left(z - \frac{1}{z}\right)\exp\left[-\frac{\sqrt{1+A_0 t}\,|x|}{2\,a_0}\left(z - \frac{1}{z}\right)\right] \qquad (3.7)$$

At last applying the Laurent expansion (3.3) to expression (3.7) one can obtain the statement of the theorem.

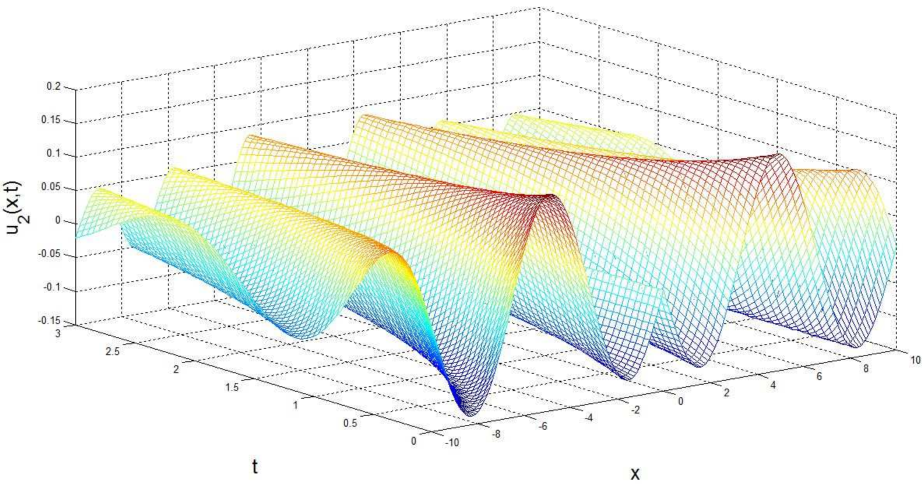
Typical graphs of the first functions (3.2) are shown on Fig. 3.2.



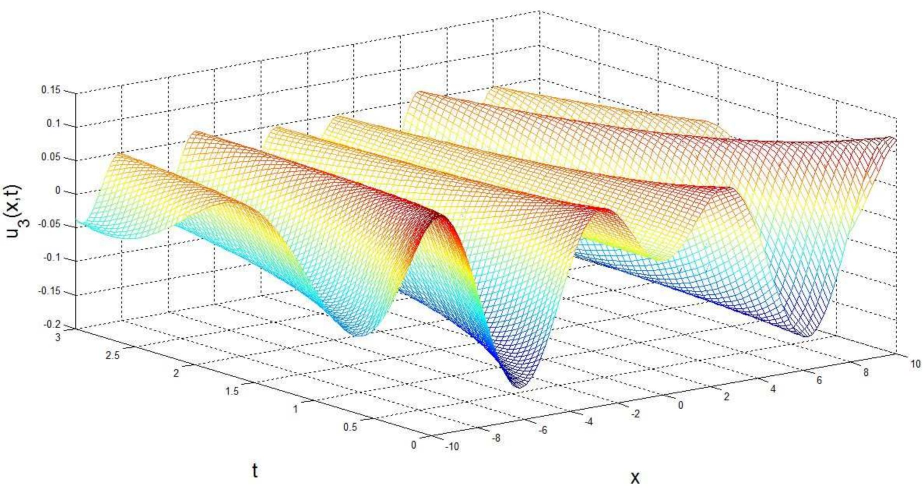
$a)$ $n = 0$



$b)$ $n = 1$



c) $n = 2$



d) $n = 3$

Рис. 3.2. Пространственно-временная эволюция первых функций $u_n(x,t)$ при $A_0 = 1$ и $a_0 = 1$ (колебательный режим): *a*) $n = 0$; *b*) $n = 1$; *c*) $n = 2$; *d*) $n = 3$
Fig 3.2. Spatiotemporal evolution of the first functions $u_n(x,t)$ under $A_0 = 1$ and $a_0 = 1$ (oscillatory behavior): *a*) $n = 0$; *b*) $n = 1$; *c*) $n = 2$; *d*) $n = 3$

3.2. Exact solution with monotone behavior

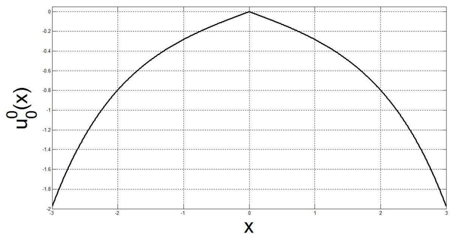
Let us consider the following vector $\{u_n^0(x)\}_{n=-\infty}^{n=+\infty}$ of initial conditions:

$$u_n^0(x) = (-1)^{n+1} \frac{A_0}{4\,a_0} \left[I_{n+1}\left(\frac{|x|}{a_0}\right) + I_{n-1}\left(\frac{|x|}{a_0}\right)\right], \quad n \in \mathbb{N},$$

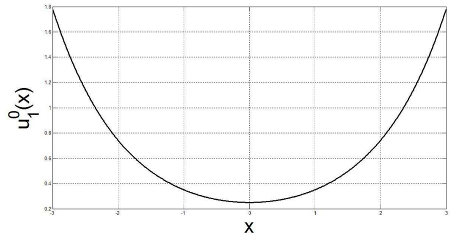
$$u_0^0(x) = -\frac{A_0}{2\,a_0}\,I_1\left(\frac{|x|}{a_0}\right), \qquad u_{-n}^0(x) = u_n^0(x)\,, \tag{3.8}$$

where $I_n(\zeta)$ are modified Bessel functions and A_0 , $a_0 > 0$.

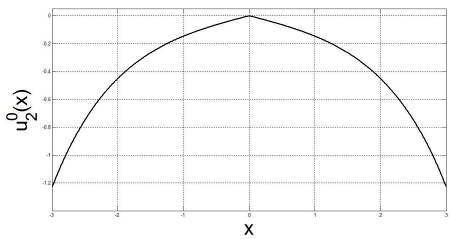
Typical graphs of the first functions (3.8) are presented on Fig. 3.3.



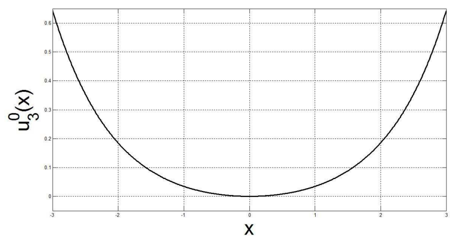
a) $n = 0$



b) $n = 1$



c) $n = 2$



d) $n = 3$

Рис. 3.3. Графики первых функций $u_n^0(x)$ при $A_0 = 1$ и $a_0 = 1$ (монотонный режим): *a)* $n = 0$; *b)* $n = 1$; *c)* $n = 2$; *d)* $n = 3$

Fig 3.3. Graphs of the first functions $u_n^0(x)$ under $A_0 = 1$ and $a_0 = 1$ (monotone behavior): *a)* $n = 0$; *b)* $n = 1$; *c)* $n = 2$; *d)* $n = 3$

The following statement is valid.

Theorem 3.2. *Exact solution of the Cauchy problem* (1.1) – (1.2) *with initial conditions* (3.8) *is equal to (*$n \in \mathbb{N}$*):*

$$u_n(x,t) = \frac{(-1)^{n+1} A_0}{4\,a_0\sqrt{1 + A_0 t}}\left[I_{n+1}\left(\frac{\sqrt{1+A_0 t}\,|x|}{a_0}\right) + I_{n-1}\left(\frac{\sqrt{1+A_0 t}\,|x|}{a_0}\right)\right],$$

$$u_0(x,t) = -\frac{A_0}{2\,a_0\sqrt{1 + A_0 t}}\,I_1\left(\frac{\sqrt{1+A_0 t}\,|x|}{a_0}\right), \qquad u_{-n}(x,t) = u_n(x,t). \tag{3.9}$$

Completely analogous to the proof of the previous theorem substitution of expressions (3.9) into the definition (2.3) of the generating function of initial condition for the input Cauchy problem and usage of the well-known Laurent series for a generating function of modified Bessel functions with integer order [8]:

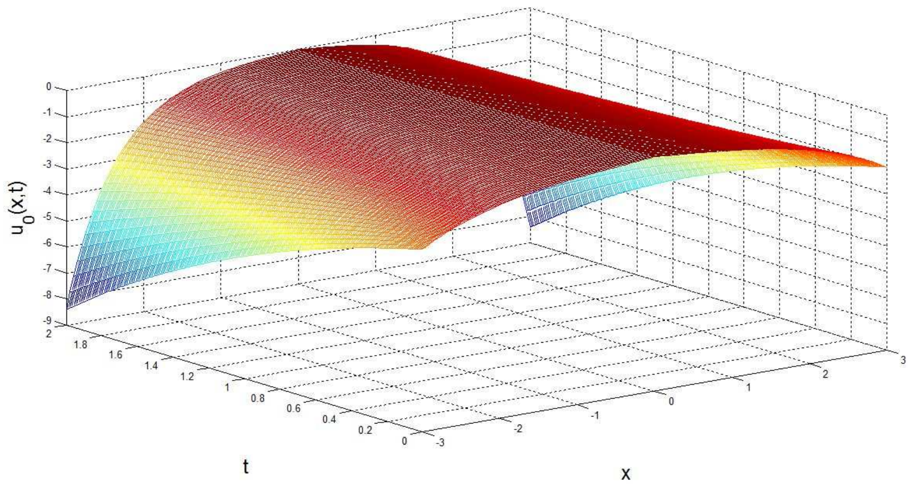
$$\exp\left[\frac{\zeta}{2}\left(z + \frac{1}{z}\right)\right] = \sum_{n=-\infty}^{+\infty} I_n(\zeta)\, z^n \tag{3.10}$$

brings us to the next formula for the generating function (2.4):

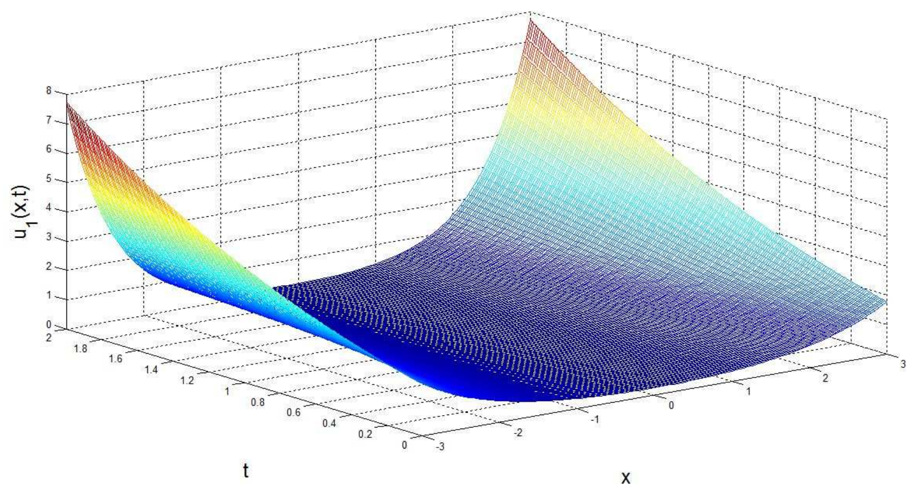
$$U(z;x,t) = \frac{A_0}{4\,a_0\,\sqrt{1+A_0 t}}\left(z+\frac{1}{z}\right)\exp\left[-\frac{\sqrt{1+A_0 t}\,|x|}{2\,a_0}\left(z+\frac{1}{z}\right)\right] \tag{3.11}$$

At last application of the Laurent expansion (3.10) to the expression (3.11) gives one the statement of the theorem.

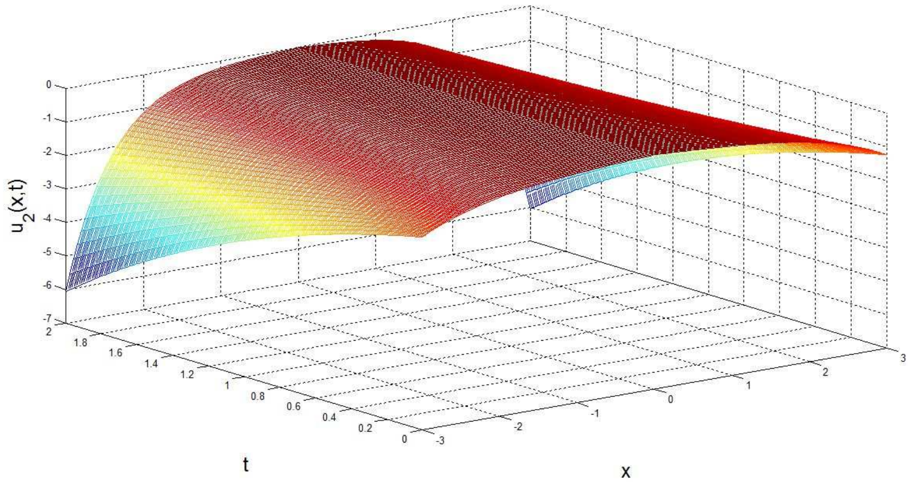
Typical graphs of the first functions (3.9) are shown on Fig. 3.4.



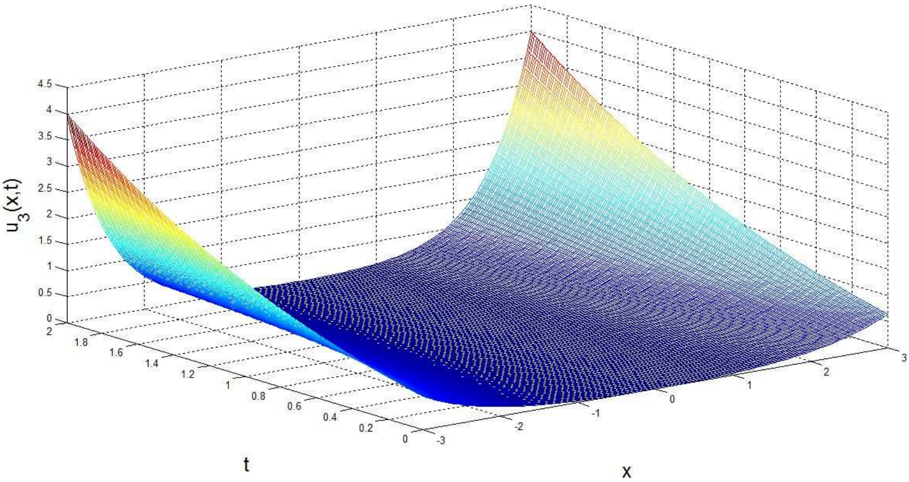
a) $n = 0$



b) $n = 1$



c) $n = 2$



d) $n = 3$

Рис. 3.4. Пространственно-временная эволюция первых функций $u_n(x,t)$ при $A_0 = 1$ и $a_0 = 1$ (монотонный режим): *a*) $n = 0$; *b*) $n = 1$; *c*) $n = 2$; *d*) $n = 3$

Fig 3.4. Spatiotemporal evolution of the first functions $u_n(x,t)$ under $A_0 = 1$ and $a_0 = 1$ (monotone behavior): *a*) $n = 0$; *b*) $n = 1$; *c*) $n = 2$; *d*) $n = 3$

4. Conclusion

In this article general form of exact solution of nonlinear countable-dimensional system of integro-differential equations (1.1) has been obtained. This result has been supplemented by construction of exact solutions of this system with both oscillatory behavior (formulas (3.2)) and monotone behavior (formulas (3.9)) corresponding to special choices of initial conditions (1.2).

Further, one can do the following nontrivial observation, namely, in the Cauchy problem (2.5) variable z is a free parameter. It means that if generating function $U(z;x,t)$ represents exact solution of the Cauchy problem (1.1) – (1.2) with initial condition representing by generating function $U^0(z;x)$ then for any $p = 2,3,4,\ldots$ generating function $U(z^p;x,t)$ represents exact solution of the Cauchy problem (1.1) – (1.2) with initial condition representing by generating function $U^0(z^p;x)$.

In other words the Laurent expansion for transformed generating function $U(z^p;x,t)$ gives one exact solution $\{\hat{u}_m(x,t)\}_{m=-\infty}^{m=+\infty}$ of the Cauchy problem (1.1) – (1.2) too as follows:

$$\hat{u}_{np}(x,t) = u_n(x,t)\,, \qquad n \in \mathbb{Z}\,, \tag{4.1}$$

where $u_n(x,t)$ are functions (3.2) or (3.9), and place between components of double-sided vector $\{\hat{u}_m(x,t)\}_{m=-\infty}^{m=+\infty}$ with numbers $n\,p$ and $n\,p+p$ are filled by zeros. The initial condition $\{\hat{u}^0_m(x)\}_{m=-\infty}^{m=+\infty}$ in this case has the same structure as the formulas (4.1).

Thus, in fact exact solutions (3.2) and (3.9) generates countable set of exact solutions of input nonlinear countable-dimensional system of integro-differential equations (1.1).

At last let us consider the following nonlinear integro-differential equation:

$$\frac{\partial u(x,y,t)}{\partial t} + \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u(x-\xi, y-\eta, t)\, u(\xi,\eta,t)\, d\xi\, d\eta = 0\,, \tag{4.2}$$

provided by initial condition:

$$u(x,y,0) = u^0(x,y)\,, \qquad (x,y) \in \mathbb{R}^2. \tag{4.3}$$

It is easy to see that the Cauchy problem (1.1) – (1.2) arises from the Cauchy problem (4.2) – (4.3). Indeed, quantization of plain $\mathbb{R}^2$ in y-direction with step δ generates from function $u(x,y,t)$ countable set of functions $u_n(x,t) = u(x, n\,\delta, t)$, $n \in \mathbb{Z}$. And after that change of integral in equation (4.2) by corresponding to it integral sum and further scaling $\delta\, u_n \to u_n$ ought to give rise to the input system (1.1) exactly. Hence, results of this article may be applied to the investigation of approximation of solutions of equation (4.2).

References

1. C. Cercignani, *The Boltzmann equation and its applications. Applied Mathematical Sciences. Volume 67*, Springer-Verlag, New York, 1988, xii+455 p.

2. D. Benney J., “Some properties of long nonlinear waves”, *Studies in applied mathematics*, **52**:1 (1973), 45–50.

3. N. Rautian A., “Correct solvability and exponential stability for solutions of Volterra integro-differential equations”, *Doklady mathematics*, **104**:2 (2021), 273–276.

4. S. Yüzbasi, M. Sezer, “An exponential approach for the system of nonlinear delay integro-differential equations describing biological species living together”, *Neural computing and applications*, **27** (2016), 769–779.

5. V. Lakshmikantham, M. Rama Mohana Rao, *Theory of integro-differential equations*, CRC Press, USA, 1995, 384 p.

6. K. Balachandran, J. Dauer P., “Controllability of Sobolev-type semilinear integrodifferential systems in Banach spaces”, *Journal of mathematical analysis and applications*, **217**:2 (1998), 335–348.

7. K. Balachandran, D. Park G., Y. Kwun C., “Nonlinear integrodifferential equation of Sobolev type with nonlocal conditions in Banach spaces”, *Communications of the Korean Mathematical Society*, **14**:1 (1999), 223–231.

8. H. Bateman, A. Erdelyi, *Higher transcedental functions. Volume 2*, Mc Graw-Hill Book Company, USA, 1953, 414 p.

Submitted 10.11.2022; Revised 19.12.2022; Accepted 25.02.2023

The author has read and approved the final manuscript.

Список литературы

1. Черчиньяни К. Теория и приложения уравнения Больцмана. М.: Мир. 1978. 496 с.

2. Benney D.J. Some properties of long nonlinear waves // Studies in applied mathematics. 1973. Vol. 52, issue 1. pp. 45–50.

3. Раутиан Н.А. Корректная разрешимость и экспоненциальная устойчивость решений вольтерровых интегро-дифференциальных уравнений // Доклады Российской академии наук. Математика, информатика, процессы управления. 2021. Т. 500. С. 62–66.

4. Yüzbasi, S., Sezer M. An exponential approach for the system of nonlinear delay integro-differential equations describing biological species living together // Neural computing and applications. 2016. Vol. 27. pp. 769–779.

5. Lakshmikantham V., Rama Mohana Rao M. Theory of integro-differential equations. CRC Press, USA. 1995. 384 p.

6. Balachandran, K., Dauer J.P. Controllability of Sobolev-type semilinear integrodifferential systems in Banach spaces // Journal of mathematical analysis and applications. 1998. Vol. 217, issue 2. pp. 335–348.

7. Balachandran, K., Park D.G., Kwun Y.C. Nonlinear integrodifferential equation of Sobolev type with nonlocal conditions in Banach spaces // Communications of the Korean Mathematical Society. 1999. Vol. 14, issue 1. pp. 223–231.

8. Бейтмен Г., Эрдейи А. Высшие трансцендентные функции. Том 2. Функции Бесселя, функции параболического цилиндра, ортогональные многочлены. М.: Наука. 1974. 296 с.

Поступила 10.11.2022; доработана после рецензирования 19.12.2022; принята к публикации 25.02.2023

Автор прочитал и одобрил окончательный вариант рукописи.